the motion,[11] but, since the free-surface boundary conditions used were still formally linearized, it is uncertain as to whether or not such procedures are a step in the right direction. A completely satisfactory answer to this question would seem to require a wholy numerical solution.

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Motion of the Center of Gravity of a Variable-Mass Body

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Nomenclature

B = body of variable mass moving in space
O' = a point fixed in space
O = a point fixed in the body B (origin of body-axes system)
G = center of gravity of B; G changes its position with respect to O as the mass varies
dm = element of mass of body B located at a point P fixed in the body
$\mathbf{R}$ = O'P
$\mathbf{r}$ = OP
$\mathbf{R}^{(O)}$ = O'O
$\mathbf{R}^{(G)}$ = O'G
$\mathbf{r}^{(G)}$ = OG
$\mathbf{F}$ = external force acting on the body
$\mathbf{K}$ = reactive force acting on the body, produced by the mass ejection
ω = angular velocity of the body B
d/dt = derivative with respect to a fixed point O_1
$\delta/\delta t$ = derivative with respect to a point O, moving with the body
M = $\int dm$, the total mass of the body B at the moment under consideration

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Mass is continuously ejected from some portion on the surface of body B. Mass is ejected with a nonzero velocity relative to the point O, and consequently the reactive forces are produced. It is assumed that ejected mass, after its separation from the body, does not affect in any way the motion of the body.

Because of the mass ejection, the center of gravity G is displaced relative to the point O. The objective of this paper is to derive the equation of motion for the center of gravity G.

The equation of motion for the body B can be expressed in the following form:

$$\int (d^2\mathbf{R}/dt^2)dm = \mathbf{F} + \mathbf{K} \tag{1}$$

where the integration is extended over the mass of the body at the time t.

For any arbitrary point P of the body B, we can write

$$d^2\mathbf{R}/dt^2 = (d^2\mathbf{R}^{(0)}/dt^2) + \boldsymbol{\omega} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \tag{2}$$

or, integrating over the mass of the body B, we can write

$$\int (d^2\mathbf{R}/dt^2)dm = \int (d^2\mathbf{R}^{(0)}/dt^2)dm + \int \boldsymbol{\omega} \times \mathbf{r}dm + \int \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})dm \tag{3}$$

Since

$$\int \mathbf{r}dm = M\mathbf{r}^{(G)} \tag{4}$$

Eq. (3) can be written in the form

$$\int (d^2\mathbf{R}/dt^2)dm = M[(d^2\mathbf{R}^{(0)}/dt^2) + \dot{\boldsymbol{\omega}} \times \mathbf{r}^{(G)} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}^{(G)})] \tag{5}$$

Since G is not fixed in the body B,

$$d^2\mathbf{R}^{(G)}/dt^2 = (d^2\mathbf{R}^{(0)}/dt^2) + \boldsymbol{\omega} \times \mathbf{r}^{(G)} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}^{(G)}) + 2\boldsymbol{\omega} \times (\delta\mathbf{r}^{(G)}/\delta t) + (\delta^2\mathbf{r}^{(G)}/\delta t^2) \tag{6}$$

Combining Eqs. (1, 5, and 6), we can write

$$M\frac{d^2\mathbf{R}^{(G)}}{dt^2} = \mathbf{F} + \mathbf{K} + 2M\boldsymbol{\omega} \times \frac{\delta r^{(G)}}{\delta t} + M\frac{\delta^2\mathbf{r}^{(G)}}{\delta t^2} \tag{7}$$

which represents the equation of motion for the variable center of gravity of the variable-mass body.

Flutter Characteristics of Titanium Alloys

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Nomenclature

E = modulus of elasticity
I = second moment of area
K = constant
M = bending moment
V = velocity
b = one-half of the chord at reference station
c = distance from neutral axis to outer fiber of section
w = weight
μ = mass ratio
σ = bending stress
ψ = flutter parameter ratio
ω = circular frequency

Introduction

Much has been done to perfect materials whose performance characteristics can meet the demands of high-speed flight at temperatures well above the functional range of

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